

# Q-balls with scalar charge.

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## Abstract

We consider Friedberg–Lee–Sirlin  $Q$ -balls in a (3+1)-dimensional model with vanishing scalar potential of one of the fields. The  $Q$ -ball is stabilized by the gradient energy of this field and carries scalar charge, over and beyond the global charge. The latter property is inherent also in a model with the scalar potential that does not vanish in a finite field region near the origin.

$Q$ -balls of the Friedberg–Lee–Sirlin type [1, 2, 3, 4] exist in models with the Lagrangians of the following sort:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + (\partial_\mu \chi)^*(\partial_\mu \chi) - h\phi^2 \chi^* \chi ,$$

where  $\phi$  is real scalar field whose potential  $V(\phi)$  has a minimum at  $\phi = \phi_1 \neq 0$ ,  $\chi$  is complex scalar field, and we consider theory in 3 spatial dimensions. For large enough global charge  $Q$  corresponding to the  $U(1)$ -symmetry  $\chi \rightarrow e^{i\alpha} \chi$ , the lowest energy state is a spherical  $Q$ -ball with  $\phi = 0$  inside and  $\phi = \phi_1$  outside. Its size  $R$  and energy  $E$  are determined by the balance of the kinetic energy of  $Q$  massless  $\chi$ -quanta confined in the potential well of radius  $R$  and the potential energy of the field  $\phi$  in the interior, i.e., they are found by minimizing

$$E(R) = \frac{\pi Q}{R} + \frac{4\pi}{3} R^3 V_0 , \tag{1}$$

where  $V_0 = V(0) - V(\phi_1)$ . Hence, the  $Q$ -ball parameters are

$$R = \left( \frac{Q}{4V_0} \right)^{1/4} , \quad E = \frac{4\sqrt{2}\pi}{3} Q^{3/4} V_0^{1/4} . \tag{2}$$

The  $Q$ -ball is stable provided its energy is smaller than the rest energy of  $Q$  massive  $\chi$ -quanta in the vacuum  $\phi = \phi_1$ ,  $E(Q) < m_\chi Q$ , where

$$m_\chi = \sqrt{h}\phi_1 . \quad (3)$$

Therefore, the estimate for the critical charge is  $Q_c \sim V_0/m_\chi^4$ . At small  $h$ , the energy of the region where the field  $\phi$  changes from zero to  $\phi_1$ , which is omitted in (1), is often indeed negligible even for the critical  $Q$ -ball.

In this note we address the question of what happens if the scalar potential  $V(\phi)$  identically vanishes,

$$V(\phi) = 0 ,$$

i.e.,  $\phi$  is a modulus field, whose vacuum expectation value is still non-zero,

$$\phi_{vac} = \phi_1 > 0 .$$

In similar situations in (2+1)-dimensional [5, 6] and (1+1)-dimensional [7] theories, the presence of a lump gives rise to the dynamical vacuum selection: the cloud of modulus is gradually ejected to spatial infinity, and the system relaxes to the absolute minimum of energy (this would be the state  $\phi = 0$  in our case). However, it has been pointed out [5, 7] that the vacuum selection effect does not operate in 3 or more spatial dimensions.

The reason for the absence of the vacuum selection is that the  $Q$ -ball is stabilized by the gradient energy of the modulus field  $\phi$ . To see this, let us consider a configuraion in which  $\phi(r)$  vanishes inside a sphere of radius  $R$  and gradually approaches  $\phi_1$  outside this sphere, see Fig. 1. We assume that the potential well  $h\phi^2(r)$  forces the wave function of  $\chi$ -quanta to vanish at  $r > R$ ; this assumption will be justified for large  $Q$  in the end of the calculation. Then the energy of  $Q$  massless  $\chi$ -quanta confined in the  $Q$ -ball is again equal to  $\pi Q/R$ . The field  $\phi$  is free at  $r > R$ , the minimization of its energy gives  $\Delta\phi = 0$ , and the field configuration is<sup>1</sup>

$$\phi(r) = -\frac{C}{r} + \phi_1 , \quad r > R , \quad (4)$$

where

$$C = R\phi_1 . \quad (5)$$

The parameter  $C$  is natuarally interpreted as the scalar charge of the  $Q$ -ball. Hence, the  $Q$ -balls we discuss experience long-ranged attraction mediated by the modulus field  $\phi$ .

The gradient energy of the  $Q$ -ball hair (4) is

$$E_\phi = \int_R^\infty \frac{1}{2} \left( \frac{C}{r^2} \right)^2 4\pi r^2 dr = 2\pi\phi_1^2 R . \quad (6)$$

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<sup>1</sup>In 1 or 2 spatial dimensions, no solutions to  $\Delta\phi = 0$  would tend to the prescribed value  $\phi_1$  as  $r \rightarrow \infty$ . This is the basic reason for the vacuum selection in these dimensions.

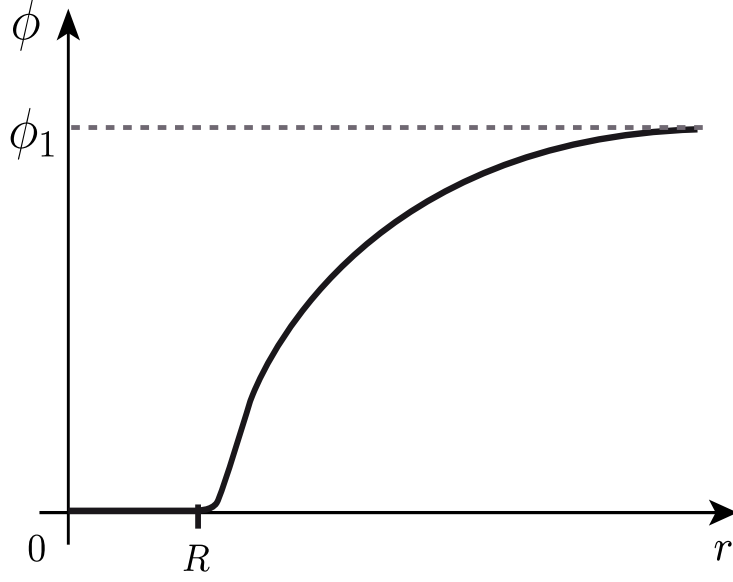


Figure 1: The profile of the field  $\phi(r)$  in the  $Q$ -ball.

Hence, the total energy of the system is given by

$$E(R) = \pi \frac{Q}{R} + 2\pi\phi_1^2 R. \quad (7)$$

By minimizing this expression with respect to  $R$ , we obtain the size, energy and scalar charge of the  $Q$ -ball,

$$R = \frac{\sqrt{Q}}{\sqrt{2}\phi_1} \quad (8)$$

$$E = 2\sqrt{2}\pi\phi_1\sqrt{Q} \quad (9)$$

$$C = \phi_1 R = \sqrt{\frac{Q}{2}} \quad (10)$$

Note that the dependence of  $R$  and  $E$  on  $Q$  is entirely different from (2). Parameters of the critical  $Q$ -ball are obtained by equating the energy (9) to the rest energy of  $Q$  quanta of the field  $\chi$  in the vacuum  $\phi = \phi_1$ , i.e.,  $E(Q_c) \sim m_\chi Q_c$ , where  $m_\chi$  is still given by (3). We obtain

$$Q_c \sim \frac{8\pi^2}{h}, \quad R_c \sim \frac{2\pi}{m_\chi}, \quad E_c \sim \frac{8\pi^2}{h} m_\chi. \quad (11)$$

We emphasize that these expressions are estimates only, since the critical size  $R_c$  is of the order of the  $\chi$ -boson mass in vacuum  $\phi = \phi_1$ , so our approximation of vanishing  $\chi$ -boson wave function at  $r > R$  is not valid for the critical  $Q$ -ball.

This approximation *is* valid for  $Q \gg Q_c$ , so the expressions (8), (9), (10) are exact in the large- $Q$  limit. To see this, let us estimate the actual spatial extent of the  $\chi$ -boson wave function in the region  $r > R$ . There, the wave function obeys

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\chi}{dr} \right) + \omega^2 \chi - m_\chi^2 \left( 1 - \frac{R}{r} \right)^2 \chi = 0 ,$$

where  $\omega = \pi/R$  is the  $\chi$ -quantum energy. It is legitimate to neglect the second term, so the WKB solution is

$$\chi \propto \exp \left[ - \int_R^r dr m_\chi \left( 1 - \frac{R}{r} \right) \right] .$$

For  $r - R \ll R$  this gives

$$\chi \propto \exp \left[ - m_\chi \frac{(r - R)^2}{2R} \right] .$$

Hence, the spatial extent is of order  $\Delta r \sim \sqrt{R/m_\chi}$ . For  $Q \gg Q_c$  we have  $R \gg m_\chi^{-1}$  and hence  $\Delta r \ll R$ , as promised.

For completeness, let us consider a model in which the potential  $V(\phi)$  vanishes at  $\phi > \phi_0$  but is non-zero at  $\phi < \phi_0$ , as shown in Fig. 2. An easily tractable case is the vacuum

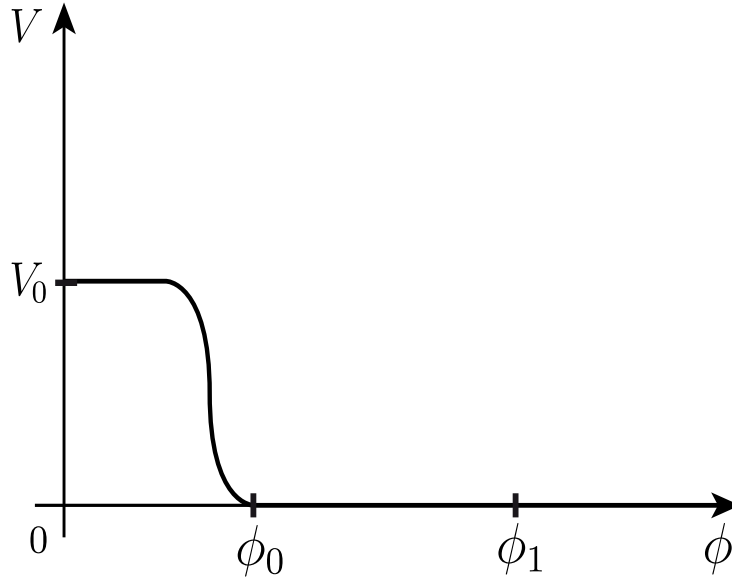


Figure 2: Scalar potential with hump near the origin.

$\phi_1 \gg \phi_0$ . In that case, there may exist a range of values of the global charge  $Q$  in which the  $Q$ -ball properties are still governed by the gradient energy of the field  $\phi(r)$  rather than its potential energy. This occurs when  $V_0 R^3 \ll \phi_1^2 R$ , see (6). We recall the result (8) and find that the gradient energy dominates over the potential energy for  $Q \ll \phi_1^4/V_0$ . The range of

global charges in question is not empty if  $\phi_1^4/V_0 \gg Q_c$ , i.e.,  $V_0 \gg m_x^2 \phi_1^2$ . If so, then in the intermediate range of global charges,

$$\frac{8\pi^2}{h} \ll Q \ll \frac{\phi_1^4}{V_0},$$

$Q$ -balls have the properties (8), (9), while for  $Q \gg \phi_1^4/V_0$  we are back to (2). For  $Q \sim \phi_1^4/V_0$  the potential and gradient energies are of the same order, and again by minimizing the energy with respect to  $R$  we obtain

$$R^2 = \frac{1}{4V_0} \left( \sqrt{4QV_0 + \phi_1^4} - \phi_1^2 \right). \quad (12)$$

It is worth noting that in either case the field profile at  $r > R$  is given by (4), i.e., the  $Q$ -ball carries non-vanishing scalar charge  $C = \phi_1 R$ , where  $R$  is given by either (8) or (2) or (12).

To summarize,  $Q$ -balls in models with  $V(\phi)$  vanishing away from the origin are rather different from the usual Friedberg–Lee–Sirlin  $Q$ -balls, since they are stabilized by gradient, rather than potential energy. Also, they carry Coulomb-like scalar hair. In this respect they are similar to the BPS monopoles. Unlike the BPS monopoles, however, the  $Q$ -balls experience long-range interactions between themselves: there is no other force to counterbalance the attraction due to the massless scalar field  $\phi$ .

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